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Commensurate-incommensurate transition in two-coupled chains of nearly half-filled electrons

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Abstract. We investigate the physical properties of two coupled chains of electrons, with a nearly half-filled band, as a function of the interchain hopping $t_\perp$ and the doping. We show that upon doping, the system undergoes a metal-insulator transition well described by a commensurate-incommensurate transition. By using bosonization and renormalization we determine the full phase diagram of the system, and the physical quantities such as the charge gap. In the commensurate phase two different regions, for which the interchain hopping is relevant and irrelevant exist, leading to a confinement-deconfinement crossover in this phase. A minimum of the charge gap is observed for values of $t_\perp$ close to this crossover. At large $t_\perp$ the region of the commensurate phase is enhanced, compared to a single chain. At the metal-insulator transition the Luttinger parameter takes the universal value $K_\rho^* = 1$, in agreement with previous results on special limits of this model.

PACS. 71.10.Hf Non-Fermi-liquid ground states, electron phase diagrams and phase transitions in model systems – 71.10.Pm Fermions in reduced dimensions (anyons, composite fermions, Luttinger liquid, etc.) – 71.30.+h Metal-insulator transitions and other electronic transitions

1 Introduction

Organic conductors Bechgaard salts, which consist of an array of one-dimensional chains, show various ordered states such as spin-Peierls state, spin density wave state and superconducting state at low temperatures [1]. Besides these states of broken symmetry, the normal state above the transition temperature exhibits a remarkable electronic state associated with a charge gap. This charge gap is supposed to originate from the electronic interactions and the commensurate band-filling (1/4 filling) of the compound [2]. Indeed in one dimension, commensurate systems are Mott insulators. Such an interpretation of the charge gap has received support from recent optical experiments [3,4] and transverse conductivity measurements [5]. In addition, since these compounds are quasi-one-dimensional systems there is a competition between the one dimensional charge gap, that tends to localize the electrons, and the interchain hopping tending to make the system three (two) dimensional. This competition could lead to a confinement-deconfinement transition responsible for the difference of behavior between the TMTTF and the TMTSF salts [2,6]. Experimentally the confinement (deconfinement) is found in TMTTF (TMTSF) salts whose interchain hopping is smaller (larger) than the charge gap [6]. From a theoretical point of view, the transition should take place when the interchain hopping renormalized by interactions is comparable with the single chain gap [2,4,7]. There is thus still considerable debate on how such a transition takes place, and what are the relevant energy scales.

Unfortunately studying an infinite number of coupled chains is extremely difficult. It is thus worth to investigate these issues on a finite number of coupled chains, *i.e.* on ladder systems [8]. By studying explicitly two-coupled chains with a half-filled band, it has been shown that the interchain hopping becomes irrelevant, in the sense of renormalization group, for a charge gap larger than the interchain hopping [9,10]. It is reasonable to identify this relevance (irrelevance) of interchain hopping with the deconfinement (confinement) for an infinite number of chains since the interchain hopping is renormalized to zero in the limit of low energy for the irrelevant case. Of course there are some differences between the simplified ladder system

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and the infinite number of chains: (i) for the ladder the confinement-deconfinement transition is in fact a crossover as far as the ground state is concerned [11,12]; (ii) in the ladder both the relevant and irrelevant cases lead to an insulating state due to a gap in the total charge excitation. Thus in the ladder to obtain a metallic state, as observed in the experimental (infinite number of chains) compounds, it would be necessary to explicitly dope the system and go away from the commensurate filling. This could be a way to mimic in this simplified model the small deviation of the commensurate filling due to warping of the Fermi surface perpendicular to chain direction [2].

Even if it is known that upon doping the ladder becomes metallic and the confined region is suppressed [13], it is yet very unclear what are the full properties of the system upon doping and what are the characteristics of such a metal-insulator transition. The purpose of the present paper is to examine these questions in the ladder system. We show that the metal-insulator transition in the ladder can be accurately described by a commensurate-incommensurate transition, and we determine its characteristics. The commensurate-incommensurate transition is well known in the classical case for one-dimension [14,15] or quasi-one-dimensional [16] system. However the quantum case which corresponds to interacting electron systems, is known only for one-dimensional case [17,18] where its connections to the Mott transition have been investigated in details [2,19,20]. In the ladder, although both the half-filled commensurate system (spin ladder) and the extremely incommensurate case have been widely investigated [21–25], comparatively little has been done on the metal-insulator transition close to half-filling. The ladder system with the umklapp scattering has been examined in the chain basis, where the incommensurate phase shows no gap in the total charge fluctuation [22]. Studies using either a mapping on a $SO(8)$ symmetric model [26,27], onto a hard core boson system [28] or in the large interchain hopping limit [29] show a drastic modification of the universal properties of the metal-insulator transition compared to the single-chain case. The universal properties close to half-filling have been checked numerically by DMRG [30]. We use here the bosonization technique and renormalization group to study the full problem as a function of the doping and the strength of the interchain hopping. This allows us to obtain the full phase diagram and in particular the interplay between the confinement-deconfinement at commensurate filling and the metal-insulator transition upon doping the ladder.

The present paper is organized as follows. In Section 2, formulation is given in terms of a bosonized phase Hamiltonian and renormalization group equations are derived by assuming scaling invariance for response functions. In Section 3, the phase diagram of the commensurate state and the incommensurate state is calculated by integrating the renormalization group equations. The charge gap is also estimated and the critical properties of the transition are given. In Section 4 a summary and a discussion of the results can be found. Technical details can be found in the Appendix.

2 Formulation

2.1 System at half-filling

We consider two-coupled chains of a quarter-filled Hubbard model with a dimerization given by [31]

$$\begin{aligned}\mathcal{H} = &-\sum_j \sum_{\sigma=\uparrow,\downarrow} \sum_{l=1,2} \left[t + (-1)^j t_{\rm d}\right] \left(c^\dagger_{j,\sigma,l}\ c_{j+1,\sigma,l} + \text{h.c.}\right) \\ &- 2t_\perp \sum_j \sum_{\sigma=\uparrow,\downarrow} \left(c^\dagger_{j,\sigma,1}\ c_{j,\sigma,2} + \text{h.c.}\right) \\ &+ U \sum_{j,l} n_{j,\uparrow,l}\, n_{j,\downarrow,l}\ , \end{aligned} \tag{1}$$

where σ ($=\uparrow,\downarrow$ or $+,-$) and l ($=1,2$) denote the spin and chain index, respectively, and $t_{\rm d}$ is the dimerization in the one-dimensional chains. After the diagonalization of the $t_{\rm d}$-term, the kinetic term is written as

$$\begin{aligned}\mathcal{H}_0 = &\sum_{k,\sigma,l} \varepsilon_k [d^\dagger_{k,\sigma,l} d_{k,\sigma,l} - u^\dagger_{k,\sigma,l} u_{k,\sigma,l}] \\ &-2t_\perp \sum_{k,\sigma} [d^\dagger_{k,\sigma,1} d_{k,\sigma,2} + \text{h.c.}] \\ &-2t_\perp \sum_{k,\sigma} [u^\dagger_{k,\sigma,1} u_{k,\sigma,2} + \text{h.c.}]\ , \end{aligned} \tag{2}$$

where $\varepsilon_k = -2\sqrt{t^2 \cos^2 ka + t_{\rm d}^2 \sin^2 ka}$ [32] with the lattice constant a and $d_{k,\sigma,l}$ ($u_{k,\sigma,l}$) is the fermion operator for the lower (upper) band on the lth chain. Note that the umklapp scattering is induced by the dimerization, which has an effect of reducing the quarter-filled band into an effectively half-filled one. Since we consider only the lower band, we use half-filling instead of quarter-filling in the present paper. By diagonalizing the interchain hopping term with the use of $a_{k,\sigma,\zeta} = (-\zeta d_{k,\sigma,1} + d_{k,\sigma,2})/\sqrt{2}$ ($\zeta = \pm$), we obtain $\mathcal{H}_0 = \sum_{k,\sigma,\zeta} \varepsilon(k,\zeta)\, a^\dagger_{k,\sigma,\zeta} a_{k,\sigma,\zeta}$ where the energy dispersion is given by

$$\varepsilon(k,\zeta) = -2\sqrt{t^2 \cos^2 ka + t_{\rm d}^2 \sin^2 ka} - 2t_\perp \zeta\ . \tag{3}$$

Here we introduce a linear dispersion, $\epsilon(k,\zeta) \to v_{\rm F}(pk - k_{{\rm F}\zeta})$ where p is the index of the branch $p = +(-)$ for the right moving (left moving) electrons and the new Fermi point is given by $k_{{\rm F}\zeta} = k_{\rm F} - \zeta 2t_\perp / v_{\rm F}$. We have neglected the $t_\perp$-dependence of the Fermi velocity. After the bosonization of electrons around the new Fermi point $k_{{\rm F}\zeta}$, we introduce the phase variables defined by [10]

$$\begin{aligned}\theta_{\nu\pm}(x) = &\frac{1}{\sqrt{2}} \sum_{q\neq 0} \frac{\pi {\rm i}}{qL} {\rm e}^{-\frac{\alpha}{2}|q| - {\rm i}qx} \sum_{k,\sigma,\zeta} f(\sigma,\zeta) \\ &\times \left(a^\dagger_{k+q,+,\sigma,\zeta} a_{k,+,\sigma,\zeta} \pm a^\dagger_{k+q,-,\sigma,\zeta} a_{k,-,\sigma,\zeta}\right), \end{aligned} \tag{4}$$

where $f(\sigma,\zeta) = 1$ (for $\nu = \rho$), σ (for $\nu = \sigma$), ζ (for $\nu = C$) and $\sigma\zeta$ (for $\nu = S$), respectively. The phase variables $\theta_{\rho+}$ and $\theta_{\sigma+}$ ($\theta_{{\rm C}+}$ and $\theta_{{\rm S}+}$), express fluctuations

of the total (transverse) charge density and spin density. They satisfy the boson commutation relation given by $[\theta_{\nu+}(x), \theta_{\nu'-}(x')] = \mathrm{i}\pi\delta_{\nu,\nu'}\,\mathrm{sgn}(x-x')$. In terms of these phase variables, the field operator, $\psi_{p,\sigma,\zeta} = L^{-1/2}\sum_k \mathrm{e}^{\mathrm{i}kx}\, a_{k,p,\sigma,\zeta}$, is given by [10]

$$\psi_{p,\sigma,\zeta}(x) = \frac{1}{\sqrt{2\pi\alpha}} \exp\left(\mathrm{i}pk_{\mathrm{F}\zeta}x + \mathrm{i}\Theta_{p,\sigma,\zeta}\right) \exp\left(\mathrm{i}\pi\Xi_{p,\sigma,\zeta}\right), \tag{5}$$

where α is of the order of the lattice constant and $\Theta_{p,\sigma,\zeta} = p/(2\sqrt{2})[\theta_{\rho+} + p\theta_{\rho-} + \sigma(\theta_{\sigma+} + p\theta_{\sigma-}) + \zeta(\theta_{\mathrm{C}+} + p\theta_{\mathrm{C}-}) + \sigma\zeta(\theta_{\mathrm{S}+} + p\theta_{\mathrm{S}-})]$. The phase factor, $\pi\Xi_{p,\sigma,\zeta}$, in equation (5), is introduced to ensure the anticommutation relation for $\psi_{p,\sigma,\zeta}$ with different p, σ and ζ.

2.2 System close to half-filling

In order to consider the system, which is slightly away from half-filling, we consider the following additional term,

$$\begin{aligned}\mathcal{H}_\mu &= -\mu \sum_{j,\sigma,l} c^\dagger_{j,\sigma,l}\, c_{j,\sigma,l} \\ &= -\mu\frac{\sqrt{2}}{\pi}\int \mathrm{d}x\, \partial_x\theta_{\rho+} \quad , \end{aligned} \tag{6}$$

where μ is the chemical potential and $\mu = 0$ corresponds to the half-filling. We apply the transformation $\sqrt{2}\theta_{\rho+} \to \sqrt{2}\theta_{\rho+} + q_0 x$ with $q_0 = 4\mu K_\rho/v_\rho$, which leads to a misfit term $q_0 x$ in the cosine term expressing the umklapp scattering. The phase Hamiltonian is written as,

$$\begin{aligned}\mathcal{H} = &\sum_{\nu=\rho,\sigma,\mathrm{C,S}} \frac{v_\nu}{4\pi}\int \mathrm{d}x\, \left[\frac{1}{K_\nu}\left(\partial\theta_{\nu+}\right)^2 + K_\nu\left(\partial\theta_{\nu-}\right)^2\right] \\ &+\frac{g_\rho}{4\pi^2\alpha^2}\int \mathrm{d}x \left[\cos\left(\sqrt{2}\theta_{\mathrm{C}+} - \frac{8t_\perp}{v_{\mathrm{F}}}x\right) + \cos\sqrt{2}\theta_{\mathrm{C}-}\right] \\ &\times\left[\cos\sqrt{2}\theta_{\mathrm{S}+} - \cos\sqrt{2}\theta_{\mathrm{S}-}\right] \\ &+\frac{g_\sigma}{4\pi^2\alpha^2}\int \mathrm{d}x \left[\cos\left(\sqrt{2}\theta_{\mathrm{C}+} - \frac{8t_\perp}{v_{\mathrm{F}}}x\right) - \cos\sqrt{2}\theta_{\mathrm{C}-}\right] \\ &\times\left[\cos\sqrt{2}\theta_{\mathrm{S}+} + \cos\sqrt{2}\theta_{\mathrm{S}-}\right] \\ &+\frac{g_u}{2\pi^2\alpha^2}\int \mathrm{d}x\, \sin\left(\sqrt{2}\theta_{\rho+} + q_0 x\right) \\ &\times\left[\cos\left(\sqrt{2}\theta_{\mathrm{C}+} - \frac{8t_\perp}{v_{\mathrm{F}}}x\right) + \cos\sqrt{2}\theta_{\mathrm{C}-}\right. \\ &\left.-\cos\sqrt{2}\theta_{\mathrm{S}+} + \cos\sqrt{2}\theta_{\mathrm{S}-}\right] \\ &+\frac{g_\perp}{2\pi^2\alpha^2}\int \mathrm{d}x\, \cos\sqrt{2}\theta_{\sigma+} \\ &\times\left[\cos\left(\sqrt{2}\theta_{\mathrm{C}+} - \frac{8t_\perp}{v_{\mathrm{F}}}x\right) - \cos\sqrt{2}\theta_{\mathrm{C}-}\right. \\ &\left.-\cos\sqrt{2}\theta_{\mathrm{S}+} - \cos\sqrt{2}\theta_{\mathrm{S}-}\right] \quad , \end{aligned} \tag{7}$$

where $v_{\rho(\sigma)} = v_{\mathrm{F}}[1+(-)\,U/\pi v_{\mathrm{F}}]^{1/2}$, $v_{\mathrm{C(S)}} = v_{\mathrm{F}}$, $K_{\rho(\sigma)} = [1+(-)\,U/\pi v_{\mathrm{F}}]^{-1/2}$, $K_{\mathrm{C(S)}} = 1$, $g_\rho = -g_\sigma = g_\perp = Ua$ and a coupling constant for the umklapp scattering is given by $g_3 = -Ua(2t_{\mathrm{d}}/t)/[1+(t_{\mathrm{d}}/t)^2]$ [32].

2.3 Renormalization group equations

By utilizing a renormalization group method, we analyze equation (7) where the nonlinear terms in equation (7) are rewritten as

$$\frac{1}{2\pi^2\alpha^2}\, g_{\nu p,\nu' p'}\int \mathrm{d}x\, \cos\sqrt{2}\,\overline{\theta}_{\nu p}\, \cos\sqrt{2}\,\overline{\theta}_{\nu' p'} \quad , \tag{8}$$

where $\overline{\theta}_{\rho+} = \theta_{\rho+} + q_0 x/\sqrt{2}$, $\overline{\theta}_{C+} = \theta_{C+} + 4\sqrt{2}t_\perp x/v_{\mathrm{F}}$ and $\overline{\theta}_{\nu p} = \theta_{\nu p}$ otherwise. The coupling constants are given by $g_{\rho+,C\pm} = \mp g_{\rho+,S\pm} = g_u$, $\pm g_{\sigma+,C\pm} = -g_{\sigma+,S\pm} = g_\perp$, $\pm g_{C\pm,S\pm} = (g_\rho + g_\sigma)/2$ and $\pm g_{C\pm,S\mp} = -(g_\rho - g_\sigma)/2$, where each coupling constant is treated in the renormalization group method. The renormalization group equations are derived from the response functions, which are given by $\langle T_\tau \exp[\mathrm{i}\theta_{\rho+}(x_1,\tau_1)]\, \exp[-\mathrm{i}\theta_{\rho+}(x_2,\tau_2)]\rangle$ [33], By making use of the scaling of the cutoff ($\alpha \to \alpha' = \alpha \mathrm{e}^{dl}$), we obtain the equations as (Appendix A),

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}l}\widetilde{t}_\perp = &\widetilde{t}_\perp - \frac{1}{8}G^2_{\rho+,C+}K_C\, F_1(8\widetilde{t}_\perp; q_0\alpha) \\ &-\frac{1}{8}\left(G^2_{\sigma+,C+} + G^2_{C+,S+} + G^2_{C+,S-}\right) K_C J_1(8\widetilde{t}_\perp), \end{aligned} \tag{9}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}l}q_0\alpha = &\, q_0\alpha - G^2_{\rho+,C+}K_\rho\, F_1(q_0\alpha; 8\widetilde{t}_\perp) \\ &-\left(G^2_{\rho+,C-} + G^2_{\rho+,S+} + G^2_{\rho+,S-}\right) K_\rho\, J_1(q_0\alpha), \end{aligned} \tag{10}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}l}K_\rho = &-\frac{1}{2}K^2_\rho\left[G^2_{\rho+,C+}\, F_0(8\widetilde{t}_\perp; q_0\alpha)\right. \\ &\left.+\left(G^2_{\rho+,C-} + G^2_{\rho+,S+} + G^2_{\rho+,S-}\right) J_0(q_0\alpha)\right], \end{aligned} \tag{11}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}l}K_\sigma = &-\frac{1}{2}\,K^2_\sigma\Big[G^2_{\sigma+,C+}\, J_0(8\widetilde{t}_\perp) + G^2_{\sigma+,C-} \\ &+G^2_{\sigma+,S+} + G^2_{\sigma+,S-}\Big] \quad , \end{aligned} \tag{12}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}l}K_C = &-\frac{1}{2}\sum_{p=\pm}\left[K^2_C\, J_0(8\widetilde{t}_\perp)\,\delta_{p,+} - \delta_{p,-}\right]\left(G^2_{Cp,S+}\right. \\ &\left.+G^2_{Cp,S-} + G^2_{\sigma+,Cp}\right) \\ &-\frac{1}{2}\sum_{p=\pm}\left[K^2_C\, F_0(8\widetilde{t}_\perp; q_0\alpha)\,\delta_{p,+}\right. \\ &\left.-J_0(q_0\alpha)\,\delta_{p,-}\right]G^2_{\rho+,Cp} \quad , \end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}l}K_S = &-\frac{1}{2}\sum_{p=\pm}\left(K^2_S\,\delta_{p,+} - \delta_{p,-}\right)\Big[G^2_{\rho+,Sp}\, J_0(q_0\alpha)\Big. \\ &\Big.+G^2_{\sigma+,Sp} + G^2_{C+,Sp}\, J_0(8\widetilde{t}_\perp) + G^2_{C-,Sp}\Big], \end{aligned} \tag{14}$$

$$\frac{\mathrm{d}}{\mathrm{d}l}G_{\nu+,C\pm} = \left(2 - K_\nu - K_C^{\pm}\right) G_{\nu+,C\pm} - G_{\nu+,S+}G_{C\pm,S+} - G_{\nu+,S-}G_{C\pm,S-}, \quad (15)$$

$$\frac{\mathrm{d}}{\mathrm{d}l}G_{\nu+,S\pm} = \left(2 - K_\nu - K_S^{\pm}\right) G_{\nu+,S\pm} - G_{\nu+,C+}\, G_{C+,S\pm}\, J_0(8\widetilde{t}_\perp) - G_{\nu+,C-}\, G_{C-,S\pm} \ , \quad (16)$$

$$\frac{\mathrm{d}}{\mathrm{d}l}G_{Cp,Sp'} = \left(2 - K_C^p - K_S^{p'}\right) G_{Cp,Sp'} - G_{\rho+,Cp}G_{\rho+,Sp'}\, J_0(q_0\alpha) - G_{\sigma+,Cp}G_{\sigma+,Sp'} \ , \quad (17)$$

where $\nu = \rho, \sigma$ and $p = \pm$. The quantities $F_0(x;y)$ and $F_1(x;y)$ are defined by

$$F_0(x;y) \equiv \frac{1}{2}[J_0(|x+y|) + J_0(|x-y|)] \ ,$$
$$F_1(x;y) \equiv \frac{1}{2}[J_1(|x+y|)\,\mathrm{sgn}(x+y) + J_1(|x-y|)\,\mathrm{sgn}(x-y)] \ ,$$

and J_n is the nth order Bessel function. In the above equations, the l-dependence is not written explicitly. The initial conditions are given by $K_\nu(0) = K_\nu$, $G_{\nu p,\nu' p'}(0) = g_{\nu p,\nu' p'}/2\pi v_\mathrm{F}$, $\widetilde{t}_\perp(0) = t_\perp/W$ with $W \equiv v_\mathrm{F}\alpha^{-1}$, and $q_0(0) = q_0$, respectively . The cutoff α can be related to the lattice constant by $\alpha = 2a/\pi$ [31], which leads to $W \simeq \pi t/\sqrt{2}$ for small t_d/t. We take a=1. The renormalization group equations for the velocity v_ν are discarded and the velocity v_ρ and v_σ are set to v_F.

In the limit of $t_\perp \to 0$ these equations reduce to those of a single chain given by

$$\frac{\mathrm{d}}{\mathrm{d}l}q_0\alpha = q_0\alpha - 4\,G_u^2\,J_1(q_0\alpha) \ , \quad (18)$$

$$\frac{\mathrm{d}}{\mathrm{d}l}G_\rho = 2\,G_u^2\,J_0(q_0\alpha) \ , \quad (19)$$

$$\frac{\mathrm{d}}{\mathrm{d}l}G_u = 2\,G_\rho\,G_u \ , \quad (20)$$

$$\frac{\mathrm{d}}{\mathrm{d}l}G_\sigma = 2\,G_\perp^2 \ , \quad (21)$$

$$\frac{\mathrm{d}}{\mathrm{d}l}G_\perp = 2\,G_\sigma\,G_\perp \ , \quad (22)$$

where $K_\rho = 1 - G_\rho$ and $K_\sigma = 1 - G_\sigma$. In the above equations, $G_u \equiv G_{\rho+,C\pm} = \mp G_{\rho+,S\pm}$, $G_\perp \equiv \pm G_{\sigma+,C\pm} = -G_{\sigma+,S\pm}$, $(G_\rho + G_\sigma) \equiv \pm 2G_{C\pm,S\pm}$ and $(G_\rho - G_\sigma) \equiv \mp 2G_{C\pm,S\mp}$. It is straightforward to derive equations (18–22) from the one-dimensional (1D) Hamiltonian given by (Appendix A)

$$\mathcal{H}_\mathrm{1D} = \frac{v_\rho}{4\pi}\int \mathrm{d}x\,\left[\frac{1}{K_\rho}\left(\partial_x\theta_+\right)^2 + K_\rho\left(\partial_x\theta_-\right)^2\right] + \frac{v_\sigma}{4\pi}\int \mathrm{d}x\,\left[\frac{1}{K_\sigma}\left(\partial_x\phi_+\right)^2 + K_\sigma\left(\partial_x\phi_-\right)^2\right] - \frac{\mu}{\pi}\int \mathrm{d}x\ \partial_x\theta_+$$

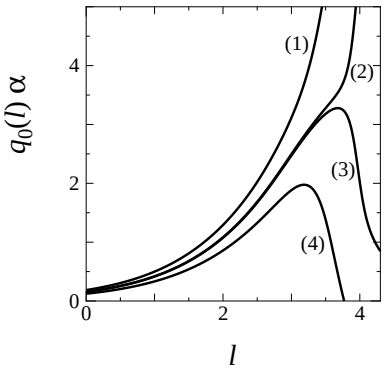

Fig. 1. The scaling flows of $q_0(l)\alpha$ with fixed $\mu/t = 0.15$ (1), 0.126 (2), 0.125 (3) and 0.1 (4) for $U/t = 5$, $t_\mathrm{d}/t = 0.05$ and $t_\perp/t = 0.1$.

$$+\frac{g_u}{2\pi^2\alpha^2}\int \mathrm{d}x\ \cos 2\theta_+ + \frac{g_\perp}{2\pi^2\alpha^2}\int \mathrm{d}x\ \cos 2\phi_+ \ , \quad (23)$$

where $\theta_\pm$ and $\phi_\pm$ are the phase variables expressing the charge and spin fluctuations, respectively [34].

3 Commensurate and incommensurate states

We examine the commensurate-incommensurate transition by solving the renormalization group equations numerically. The dimerization is taken as $t_\mathrm{d}/t = 0.05$ where such a choice does not change qualitatively the results as seen later. The scaling quantity l is related to energy ω and/or temperature by $l = \ln(W/\omega) = \ln(W/T)$.

Equation (10) shows that the quantity $q_0(l)$ in the absence of the interaction increases as $q_0(l) = q_0\mathrm{e}^l$, while the increase of $q_0(l)$ is suppressed by the presence of $G_{\rho+,C\pm}$ and $G_{\rho+,S\pm}$ coming from the umklapp scattering g_u. In Figure 1, $q_0(l)$ are shown with several choices of μ/t for $U/t = 5$, $t_\mathrm{d}/t = 0.05$ and $t_\perp/t = 0.1$. For $\mu/t = 0.15$ (curve (1)), the misfit term $q_0(l)$ increases rapidly and becomes relevant. In this case, the umklapp scattering can be neglected since the Bessel functions $J_0(q_0\alpha)$ and $J_1(q_0\alpha)$ in equations (9–17) become small due to large $q_0\alpha$. The quantity $K_\rho(l)$ at the fixed point takes a finite value due to the relevant $q_0\alpha$, and then the incommensurate state is obtained. For smaller value of the chemical potential given by $\mu/t = 0.1$ (curve (4)), the quantity $q_0(l)\alpha$ has a maximum and reduces to zero with increasing l, showing the irrelevant $q_0(l)\alpha$. Such a behavior of $q_0(l)\alpha$ leads to the commensurate state, which gives an insulating state due to the zero value of $K_\rho(l)$ in the limit of large l. We note that the commensurate state corresponds to that of a half-filled case. The commensurate-incommensurate transition occurs at the critical value given by $\mu/t = \mu_\mathrm{c}/t(\simeq 0.125)$ (curve (3)).

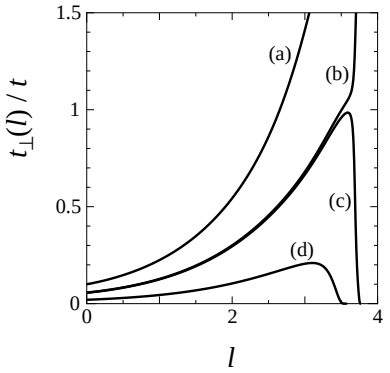

Fig. 2. The scaling flows of $t_\perp(l)/t$ with fixed $t_\perp/t = 0.1(a)$, 0.057 (b), 0.056 (c) and 0.02 (d) for $\mu/t = 0.1$, $U/t = 5$ and $t_\mathrm{d}/t = 0.05$.

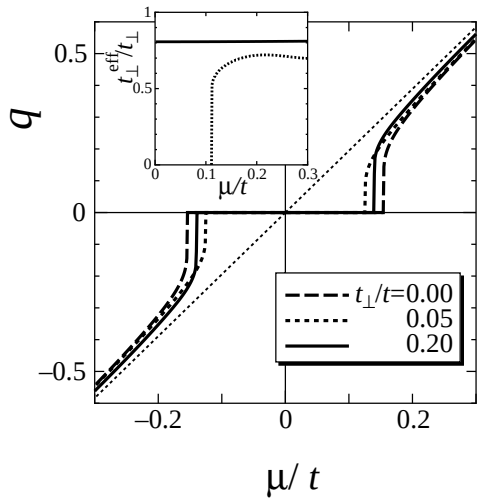

Fig. 3. The effective quantity q *vs.* the chemical potential μ/t with fixed $t_\perp/t = 0$, 0.05 and 0.2 for $U/t = 5$ and $t_\mathrm{d}/t = 0.05$. The thin dotted line denotes q_0. The inset shows $t_\perp^{\mathrm{eff}}/t_\perp$, for $t_\perp/t = 0.05$ (dotted curve) and 0.20 (dashed curve).

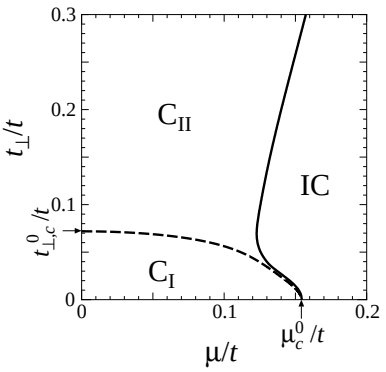

Fig. 4. The phase diagram of commensurate (C_I and C_{II}) and incommensurate (IC) states on the plane of μ/t and $t_\perp/t$ for $U/t = 5$ and $t_\mathrm{d}/t = 0.05$. The region C_I (C_{II}) denotes "Confined phase" ("D-Mott phase"), *i.e.*, the commensurate state with the irrelevant (relevant) interchain hopping. The region IC corresponds to "Luther-Emery liquid". The arrow on the horizontal (vertical) axis shows the critical value μ_c^0/t ($t_{\perp,\mathrm{c}}^0/t$) separating the respective regions.

3.1 Half-filled case

It is known that a transition from the relevant interchain hopping to the irrelevant one occurs with increasing $t_\perp$ at half-filling [9,10]. We show that such a result is also obtained for $\mu \neq 0$, which still leads to the commensurate state. In Figure 2, the quantity $t_\perp(l)/t$ is calculated for the commensurate state with $\mu/t = 0.1$. With decreasing the interchain hopping as $t_\perp/t = 0.1$ (a), 0.057 (b), 0.056 (c) and 0.02 (d), the hopping changes from a divergent behavior to a convergent one with zero value. One finds the relevant interchain hopping for large $t_\perp$ (curve (a)) but irrelevant one for small $t_\perp$ (curve (d)). The behavior which separates these two cases is obtained at a critical value of $t_\perp = t_{\perp,\mathrm{c}}$ (curve (c)). For a relevant interchain hopping, one obtains an insulating phase with spin gap excitations which is called "D-Mott" phase [26] with "C0S0" phase. The notation CnSm denotes a state with n massless charge mode and m massless spin mode. Such a spin gap is also known in the $SO(8)$ Gross-Neveu model [26], which includes an additional term given by $g_{\rho+,\sigma+} \sin\sqrt{2}\theta_{\rho+} \cos\sqrt{2}\theta_{\sigma+}$. For an irrelevant interchain hopping, one can also expect a state with the spin gap due to the following fact. The phase has all charge excitations gapped, and because of such generated Heisenberg exchange ($J \simeq t_\perp^2/\Delta_\rho$) [11] is equivalent to a spin ladder. Such a system is known to have all spin excitations gapped. We note that, for two-coupled chains, these states undergo a crossover since both states exhibit C0S0 phase and there is no clear distinction between these two states [11,12]. However we may identify the irrelevant (relevant) $t_\perp$ as confinement (deconfinement) since such a result could be expected for the case of many chains [35] and higher dimension.

3.2 Commensurate-incommensurate transition

Based on Figures 1 and 2, we examine quantities q and $t_\perp^{\mathrm{eff}}$, which express effective quantities of q_0 and $t_\perp$ in the presence of the interaction and the chemical potential. These quantities are estimated from $q\alpha = \exp[-l_q] \times 4$ with $q_0(l_q)\alpha = 4$, and $t_\perp^{\mathrm{eff}} = t\exp[-l_1]$ with $t_\perp(l_1)/t = 1$. One finds that $q = q_0$ and $t_\perp^{\mathrm{eff}} = t_\perp$ for the noninteracting case. In the present analysis, the effective quantity q is related to the carrier density n by $n = q/\pi$ (Appendix A). In Figure 3, the μ-dependence of q is shown with fixed $t_\perp/t = 0$, 0.05 and 0.2 for $U/t = 5$ and $t_\mathrm{d}/t = 0.05$. The quantity q, which is suppressed by the umklapp scattering, becomes zero in the commensurate phase. The thin dotted line denotes $q\,(= q_0)$ in the absence of the umklapp scattering. The critical value μ_c for the commensurate-incommensurate transition is not monotonic as a function of $t_\perp$ since μ_c for $t_\perp/t = 0$ is larger (smaller) than that for $t_\perp/t = 0.05$ (0.2). The explicit $t_\perp$-dependence of μ_c is evaluated in Figure 4. In the inset of Figure 3, the effective interchain hopping normalized by $t_\perp$ is shown

for $t_\perp/t = 0.05$ (dotted curve) and 0.2 (solid curve) where $t_\perp^{\rm eff}/t_\perp$ is symmetric with respect to $\mu = 0$. For $t_\perp/t = 0.05$, $t_\perp^{\rm eff}/t_\perp$ becomes zero for small $\mu/t(\lesssim 0.11)$ and exhibits the irrelevant interchain hopping while the interchain hopping for $t_\perp/t = 0.2$ shows always relevant one.

From the results of Figure 3, we examine the boundary between the commensurate state (C_I and C_{II}) and the incommensurate state (IC) and also the boundary between the relevant interchain hopping and irrelevant one. In Figure 4, the phase diagram of these states is shown for $U/t = 5$ and $t_d/t = 0.05$. The solid curve denotes the boundary between the incommensurate state and the commensurate state. The incommensurate state corresponds to the metallic state. The commensurate state is the (Mott-) insulating state where the dashed curve in the commensurate state denotes the boundary between the relevant interchain hopping ("D-Mott phase") [11] and the irrelevant one ("Confined phase"). The critical value shown by the arrow on the $t_\perp$-axis is given by $t_{\perp,c}^0/t \simeq 0.072$ [31, 35]. The critical value shown by the arrow on the μ-axis is given by $\mu_c^0/t \simeq 0.154$, at which the solid curve merges with the dashed curve. The interchain hopping is always relevant in the incommensurate phase since the umklapp scattering becomes irrelevant in the limit of low energy as seen from equation (7). The incommensurate phase shows no gap in the total charge fluctuation due to the relevant μ [22], while the gaps still exist for other fluctuations due to the relevant $t_\perp$. The incommensurate state corresponds to a "C1S0" d-wave superconductivity [25] and is called "Luther-Emery liquid" [29]. The solid curve corresponding to the critical value μ_c shows non-monotonic behavior as a function of $t_\perp$. With increasing $t_\perp$, μ_c has a minimum at $t_\perp/t \simeq 0.07$ and becomes larger than μ_c^0. The decrease of μ_c for small $t_\perp/t(\lesssim 0.07)$ originates in the fact that the effect of the umklapp scattering becomes weakened for finite $t_\perp$ due to the misfit $(8t_\perp/v_F)x$ in g_u-term of equation (7).

For large $t_\perp$ and small g_u (g_u is controlled by t_d which is small here) we can explain the behavior by a qualitative analysis of the renormalization group equations. The strong interchain hopping between the two interacting chains opens a gap in all sectors except for the total charge sector. We may define a scale l_1 where the couplings in the σ, C, S sectors have reached a value of order one. This scale will depend on $t_\perp$ and the value of the bare interaction but not on g_u. Up to l_1, g_u will renormalize by some finite multiplicative constant that will not affect the asymptotic dependence of the charge gap. However above l_1 the scaling dimension of g_u is $2-K_\rho$, instead of $2-2K_\rho$ if $t_\perp = 0$. Indeed $\cos(\sqrt{2}\theta_{S+})$ and $\cos(\sqrt{2}\theta_{C-})$ acquire a mean value (with opposite signs) so that the umklapp term reduces to:

$$g_u \sin\sqrt{2}\theta_{\rho+}\left(\langle\cos\sqrt{2}\theta_{C-}\rangle - \langle\cos\sqrt{2}\theta_{S+}\rangle\right) \quad . \qquad (24)$$

As a first consequence this means that the power law dependence of the gap for asymptotically small umklapp scattering is enhanced by interchain hopping from $\Delta_\rho^{\rm 1D} \propto g_u^{1/(2-2K_\rho)}$ to $\Delta_\rho \propto g_u^{1/(2-K_\rho)}$. The second physical consequence is for the commensurate-incommensurate transition. In the absence of chemical potential of equation (6), the operator g_u in equation (24) would be relevant when $K_\rho^{\mu=0} < 2$. As is standard for the commensurate-incommensurate transition [2,25], the addition of a chemical potential of equation (6) leads to a universal value of the Luttinger exponent close to zero doping of $K_\rho^* = K_\rho^{\mu=0}/2 = 1$. It is also easy to see that the charge excitations connecting two minima of the potential, equation (24), correspond to a charge +2. We thus recover quite generally the results that were established in the various previous limits [27–30].

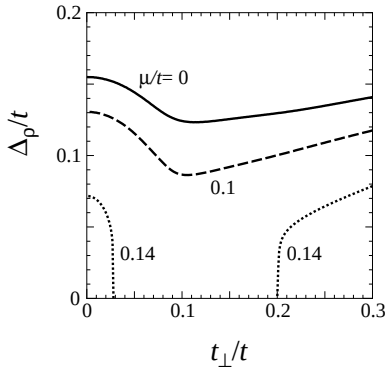

Fig. 5. The $t_\perp$-dependence of the charge gap Δ_ρ/t for fixed $\mu/t = 0$, 0.1 and 0.14 for $U/t = 5$ and $t_d/t = 0.05$.

Next, we estimate the magnitude of the charge gap on the plane of μ/t and $t_\perp/t$ of Figure 4. In a way similar to the previous calculations in the case for $\mu = 0$ [9,10], the charge gap Δ_ρ is calculated from the renormalization group flow where $\Delta_\rho/t = W\exp[-l_\Delta]$ and l_Δ is defined by $K_\rho(l_\Delta) \equiv 1/2$. We set $\Delta_\rho = 0$ for $K_\rho(\infty) > 1/2$. In Figure 5, the $t_\perp$-dependence of Δ_ρ is shown for $\mu/t = 0$, 0.1 and 0.14. When $t_\perp$ increases, the charge gap with $\mu = 0$ decreases and has a minimum at $t_\perp/t = 0.154$. A similar dependence is found in the $t_\perp$-dependence of μ_c in Figure 4 where the location for the minimum is the same within the numerical accuracy. The identification of the charge gap for the commensurate case $\Delta_\rho(\mu = 0)$ with μ_c can be expected for the commensurate-incommensurate transition on general grounds. Indeed the charge gap is the smallest energy one must pay to inject an excitation which carries a charge. When the chemical potential reaches the charge gap, charged excitations are injected in the system, the system is incommensurate. The non-monotonic dependences of Δ_ρ on $t_\perp$ indicate a crossover from the irrelevant interchain hopping to the relevant one with increasing $t_\perp$. For $\mu/t = 0.1$, Δ_ρ is suppressed but is still similar to that of $\mu = 0$. However the $t_\perp$-dependence of Δ_ρ for $\mu/t = 0.14$ is qualitatively different from others. The metallic state with $\Delta_\rho/t = 0$ appears in the interval region of $0.028 < t_\perp/t < 0.2$. This region corresponds to the incommensurate state in Figure 4. We note that numerical integration of the renormalization group flow shows that Δ_ρ as a function of μ decreases monotonically to zero at the commensurate-incommensurate transition,

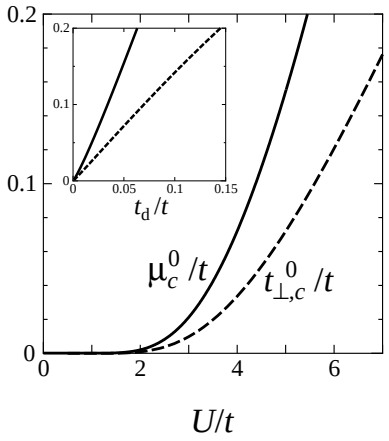

Fig. 6. The U/t-dependence of $\mu_{\rm c}^0$ (solid curve) and $t_{\perp,{\rm c}}^0$ (the dashed curve) for $t_{\rm d}/t = 0.05$, which are shown by the arrows of the horizontal axis and the vertical axis in Figure 4, respectively. The inset shows the corresponding $t_{\rm d}$-dependence for $U/t = 5$.

in agreement with the annihilation of the gap expected for a commensurate-incommensurate transition.

Finally we examine the U and $t_{\rm d}$ dependences of the critical values, $\mu_{\rm c}^0$ and $t_{\perp,{\rm c}}^0$, which are shown by the arrows in Figure 4. In Figure 6 the U-dependences of $\mu_{\rm c}^0$ and $t_{\perp,c}$ are shown by the solid curve and the dashed curve respectively for $t_{\rm d}/t = 0.05$. In the inset, the $t_{\rm d}$-dependences of the corresponding quantities are shown by solid and dashed curves for $U/t = 5$. These U and $t_{\rm d}$-dependences show that $\mu_{\rm c}^0/t_{\perp,{\rm c}}^0 \simeq 2.2$ for the present choice of parameters. Thus a rescaled phase diagram, which is independent of U and $t_{\rm d}$, can be obtained from Figure 4 by using the rescaled variables $\mu/\mu_{\rm c}^0$ and $t_\perp/t_{\perp,{\rm c}}^0$. Our renormalization group procedure correctly gives a value for $\mu_{\rm c}^0$ identical, within the scale of Figure 4, with $\Delta_\rho^{\rm 1D}$. The quantity $\Delta_\rho^{\rm 1D}$ denotes Δ_ρ of the single chain where $\Delta_\rho^{\rm 1D} \simeq W(g_u/W)^{1/(2-2K_\rho)}$ for small g_u [4]. We note that the boundary between a relevant $t_\perp$ and an irrelevant $t_\perp$ is determined more accurately by the competition between $t_\perp^{\rm eff,0}$ and $\Delta_\rho^{\rm 1D}$ where $t_\perp^{\rm eff,0}$ denotes the effective interchain hopping energy for $t_{\rm d} = \mu = 0$ [35,36]. The quantity $t_\perp^{\rm eff,0}$ is given analytically by $t_\perp^{\rm eff,0} = t_\perp(t_\perp/t)^{\alpha_0/(1-\alpha_0)}$ with $\alpha_0 = (K_\rho + K_\rho^{-1} + K_\sigma + K_\sigma^{-1} - 4)/4$ [7,37,38].

4 Summary and discussion

In the present paper, we investigated the properties of two-coupled chains with interchain electron hopping and a filling close to half-filling. By using bosonization and a renormalization group method we obtained the full phase diagram of the system. There is a metal-insulator transition for a critical value $\mu_{\rm c}$ of the chemical potential, describable by a commensurate-incommensurate phase transition on the bosonized Hamiltonian. The critical value $\mu_{\rm c}$, shown in Figure 4, exhibits a non-monotonic dependence on the perpendicular hopping $t_\perp$. The minimum of $\mu_{\rm c}$ occurs for values of $t_\perp$ close to the ones for which a confinement deconfinement crossover takes place for the commensurate case. For large $t_\perp$ the relevance of interchain hopping reinforces the commensurate character of the system leading to an enhanced commensurability gap. The crossover line separating the regions of irrelevant and relevant interchain hopping (confinement-deconfinement line) merges with the boundary between the commensurate state and the incommensurate state at $\mu = \mu_{\rm c}^0$, the critical chemical potential for a single chain ($t_\perp = 0$). We found that the phase diagram of Figure 4 becomes almost independent of parameters such as interactions and dimerization when expressed in terms of the rescaled variables $\mu/\mu_{\rm c}^0$ ($t_\perp/t_{\perp,{\rm c}}^0$). In addition, given the form for the Hamiltonian, we could show that at the limit of small doping, the Luttinger liquid parameter takes the universal value $K_\rho^* = 1$, thereby confirming the results [26–30] obtained on specific limits of the model.

Finally let us discuss the interchain exchange interactions in the commensurate confined phase (irrelevant interchain hopping). Even when irrelevant, the interchain hopping generates two particles and particle hole hopping [7]. Within the present formalism, the two particle interchain hopping can be taken into account by starting from the chain. The Hamiltonian corresponding to the umklapp scattering, $\mathcal{H}_u$, in equation (7) is rewritten as

$$\begin{aligned}\mathcal{H}_u = {} & g_u^{\rm 1D}\sum_{p,l}\int {\rm d}x\ \psi_{p,\uparrow,l}^\dagger\,\psi_{-p,\uparrow,l}\,\psi_{p,\downarrow,l}^\dagger\,\psi_{-p,\downarrow,l} \\ & + J_{uz}\sum_{p}\int {\rm d}x\ \left[\psi_{p,\uparrow,1}^\dagger\,\psi_{-p,\uparrow,1}\,\psi_{p,\downarrow,2}^\dagger\,\psi_{-p,\downarrow,2} + {\rm h.c.}\right] \\ & + J_{u\perp}\sum_{p}\int {\rm d}x\ \left[\psi_{p,\uparrow,1}^\dagger\,\psi_{-p,\downarrow,1}\,\psi_{p,\downarrow,2}^\dagger\,\psi_{-p,\uparrow,2} + {\rm h.c.}\right] \\ & + g_u^{pt}\sum_{p}\int {\rm d}x\ \left[\psi_{p,\uparrow,1}^\dagger\,\psi_{p,\downarrow,1}^\dagger\,\psi_{-p,\downarrow,2}\,\psi_{-p,\uparrow,2} + {\rm h.c.}\right],\end{aligned} \tag{25}$$

where $\psi_{p,\sigma,l} = L^{-1/2}\sum_k {\rm e}^{{\rm i}kx} d_{k,p,\sigma,l}$ with $l = 1,2$ denoting the chain index and $p = +(-)$ corresponding to the right (left) moving electrons. The coupling constants are given by $g_u^{\rm 1D} = (g_{\rho+,C+} + g_{\rho+,C-} - g_{\rho+,S+} + g_{\rho+,S-})/4$, $J_{uz} = (g_{\rho+,C+} - g_{\rho+,C-} - g_{\rho+,S+} - g_{\rho+,S-})/4$, $J_{u\perp} = -(g_{\rho+,C+} - g_{\rho+,C-} + g_{\rho+,S+} + g_{\rho+,S-})/4$ and $g_u^{pt} = (g_{\rho+,C+} + g_{\rho+,C-} + g_{\rho+,S+} - g_{\rho+,S-})/4$, where $g_u^{\rm 1D}$ and J_{uz} ($J_{u\perp}$) denote the intrachain umklapp scattering and the interchain umklapp exchange, and g_u^{pt} denotes the pair tunneling between chains. Other terms in the Hamiltonian can be rewritten in a similar form. The initial values of the interchain exchange and that of pair tunneling are zero since the initial values of the umklapp scattering in the renormalization group equations are given by $g_{\rho+,C+} = g_{\rho+,C-} = -g_{\rho+,S+} = g_{\rho+,S-} = g_u$. Both of these interactions are generated by the renormalization.

It would be a interesting problem to investigate, by taking the higher order terms in the renormalization group, the consequences of these terms on the full phase diagram investigated in this paper.

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Appendix A: Renormalization group equations

In this section, we derive the renormalization group equation for μ.

First, we treat the system with a single chain where the phase Hamiltonian is given by

$$\mathcal{H}_{1\mathrm{D}} = \frac{v_\rho}{4\pi}\int \mathrm{d}x\,\left[\frac{1}{K_\rho}\left(\partial_x\theta_+\right)^2 + K_\rho\left(\partial_x\theta_-\right)^2\right] + \frac{g_u}{2\pi^2\alpha^2}\int \mathrm{d}x\,\cos\left(2\theta_+ + q_0 x\right)\ . \tag{26}$$

The quantities v_ρ and K_ρ are the same as two-coupled chains, and $[\theta_+(x), \theta_-(x')] = \mathrm{i}\pi\mathrm{sgn}(x-x')$ [34] and $q_0 = (4K_\rho/v_\rho)\mu$. Here the expectation value of the carrier density, n, can be evaluated as

$$n = \frac{2K_\rho}{\pi v_\rho}\mu + \frac{1}{\pi}\frac{T}{L}\int \mathrm{d}x\ \mathrm{d}\tau\ \langle\partial_x\theta_+\rangle\ , \tag{27}$$

where the factor $2K_\rho/\pi v_\rho$ in the first term of r.h.s. corresponds to the compressibility in the absence of the umklapp scattering. The second term of equation (27) can be calculated as follows,

$$\begin{aligned}&\int \mathrm{d}x\ \mathrm{d}\tau\ \langle\partial_x\theta_+\rangle\\ &= \frac{1}{Z}\mathrm{Tr}\left[\frac{\partial}{\partial\lambda}\exp\left(-\int \mathrm{d}\tau\,\mathcal{H}_{1\mathrm{D}} + \lambda\int \mathrm{d}x\ \mathrm{d}\tau\,\partial_x\theta_+\right)\Big|_{\lambda=0}\right]\\ &= \frac{4G_u}{\alpha^2}K_\rho\int \mathrm{d}x\ \mathrm{d}\tau\, x\,\langle\sin\left(2\theta_+ + q_0 x\right)\rangle\ ,\end{aligned} \tag{28}$$

where $Z = \mathrm{Tr}\exp(-\int \mathrm{d}\tau\,\mathcal{H}_{1\mathrm{D}})$ and $G_u = g_u/(2\pi v_\rho)$. In equation (28), the new phase variable $\widetilde{\theta}_+ (= \theta_+ - 2\pi K_\rho\lambda x/v_\rho)$ has been introduced and rewritten as $\widetilde{\theta}_+ \to \theta_+$. Then equation (27) leads

$$n = \frac{1}{2\pi}q_0 + \frac{4}{\pi\alpha^2}G_u K_\rho\frac{T}{L}\int \mathrm{d}x\ \mathrm{d}\tau\, x\,\langle\sin\left(2\theta_+ + q_0 x\right)\rangle\cdot \tag{29}$$

Here it is worthwhile noting that equation (29) is compared with

$$\mu = \frac{\pi v_\rho}{2K_\rho}n - \frac{2v_\rho}{\alpha^2}G_u\frac{T}{L}\int \mathrm{d}x\ \mathrm{d}\tau\, x\,\langle\sin\left(2\theta_+ + 2\pi n x\right)\rangle\,, \tag{30}$$

which is obtained by using the Legendre transformation, *i.e.*, by calculating the value of the chemical potential at fixed carrier density n [19,33].

After a straightforward calculation of equation (29), one finds, up to the lowest order of G_u,

$$n = \frac{1}{2\pi}q_0 - \frac{2}{\pi\alpha}G_u^2 K_\rho\int\frac{\mathrm{d}r}{\alpha}\left(\frac{r}{\alpha}\right)^{2-4K_\rho}J_1(q_0 r). \tag{31}$$

By assuming that the quantity n is simply scaled as $n(l) = n\,\mathrm{e}^l$ [33] with the transform $\alpha \to \alpha\,\mathrm{e}^l$, the renormalization group equation for q_0 is obtained as,

$$\frac{\mathrm{d}}{\mathrm{d}l}q_0\alpha = q_0\alpha - 4G_u^2\,K_\rho\,J_1(q_0\alpha)\ . \tag{32}$$

The renormalization group equations for K_ρ and G_u can be obtained in a way similar to reference [19] as

$$\frac{\mathrm{d}}{\mathrm{d}l}K_\rho = -2\,G_u^2\,K_\rho^2\,J_0(q_0\alpha)\ , \tag{33}$$

$$\frac{\mathrm{d}}{\mathrm{d}l}G_u = (2-2K_\rho)\,G_u\ . \tag{34}$$

By integrating this renormalization group equation, the effective quantity of q_0 can be estimated from $q\alpha = c\exp(-l_q)$ with $q_0(l_q)\alpha = c$, where c is numerical constant of the order of unity. One finds that the quantity q can be related to carrier density n by $n = q/(2\pi)$ from equation (31).

Next we calculate the case of two-coupled chains. The expectation value of the carrier density, n, can be evaluated as

$$n = \frac{4K_\rho}{\pi v_\rho}\mu + \frac{\sqrt{2}}{\pi}\frac{T}{L}\int \mathrm{d}x\ \mathrm{d}\tau\ \langle\partial_x\theta_{\rho+}\rangle\,, \tag{35}$$

where the first term of r.h.s. becomes twice as that in equation (27), since here we consider two chains. From a procedure similar to the single chain, equation (29) is replaced by

$$\begin{aligned}n = \ & \frac{1}{\pi}q_0 + \frac{4}{\pi\alpha^2}G_{\rho+,C+}\,K_\rho\frac{T}{L}\int \mathrm{d}x\ \mathrm{d}\tau\, x\\ &\times\left\langle\sin\left(\sqrt{2}\theta_{\rho+} + q_0 x\right)\cos\left(\sqrt{2}\theta_{C+} - \frac{8t_\perp}{v_\mathrm{F}}x\right)\right\rangle\\ &+ \frac{4}{\pi\alpha^2}G_{\rho+,C-}\,K_\rho\frac{T}{L}\int \mathrm{d}x\ \mathrm{d}\tau\, x\\ &\times\left\langle\sin\left(\sqrt{2}\theta_{\rho+} + q_0 x\right)\cos\sqrt{2}\theta_{C-}\right\rangle\\ &+ \frac{4}{\pi\alpha^2}G_{\rho+,S+}\,K_\rho\frac{T}{L}\int \mathrm{d}x\ \mathrm{d}\tau\, x\\ &\times\left\langle\sin\left(\sqrt{2}\theta_{\rho+} + q_0 x\right)\cos\sqrt{2}\theta_{S+}\right\rangle\\ &+ \frac{4}{\pi\alpha^2}G_{\rho+,S-}\,K_\rho\frac{T}{L}\int \mathrm{d}x\ \mathrm{d}\tau\, x\\ &\times\left\langle\sin\left(\sqrt{2}\theta_{\rho+} + q_0 x\right)\cos\sqrt{2}\theta_{S-}\right\rangle\cdot\end{aligned} \tag{36}$$

The coupling constants that appear in equation (36) are those including the misfit $q_0 x$ in the cosine potential of equation (7). The calculation in the lowest

order of perturbation yields,

$$\begin{aligned} n = & \frac{1}{\pi} q_0 + \frac{1}{\pi\alpha} G^2_{\rho+,C+} K_\rho \\ & \times \int \frac{\mathrm{d}r}{\alpha} \left(\frac{r}{\alpha}\right)^{2-2K_\rho-2K_C} F_1\left(q_0 r; 8t_\perp r/v_\mathrm{F}\right) \\ & + \frac{1}{\pi\alpha} G^2_{\rho+,C-} K_\rho \int \frac{\mathrm{d}r}{\alpha} \left(\frac{r}{\alpha}\right)^{2-2K_\rho-2/K_C} J_1(q_0 r) \\ & + \frac{1}{\pi\alpha} G^2_{\rho+,S+} K_\rho \int \frac{\mathrm{d}r}{\alpha} \left(\frac{r}{\alpha}\right)^{2-2K_\rho-2K_S} J_1(q_0 r) \\ & + \frac{1}{\pi\alpha} G^2_{\rho+,S-} K_\rho \int \frac{\mathrm{d}r}{\alpha} \left(\frac{r}{\alpha}\right)^{2-2K_\rho-2/K_S} J_1(q_0 r) \ , \end{aligned} \tag{37}$$

where $F_1(x;y) \equiv [J_1(|x+y|)\,\mathrm{sgn}(x+y) + J_1(|x-y|)\,\mathrm{sgn}(x-y)]/2$. The infinitesimal transform of the cutoff $\alpha \to \alpha' = \alpha\,\mathrm{e}^{dl}$ in equation (37) leads the renormalization equation, equation (10). The renormalization group equation for $t_\perp$, equation (9), can be obtained in a similar way. The equations for the other coupling constants, equations (11–17) are also obtained in a way similar to reference [10].

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